Determination of the MESSENGER spacecraft orbit by processing doppler data in the X-band

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MESSENGER (MErcury Surface, Space Environment, GEochemistry, and Ranging)

The MESSENGER spacecraft was launched into space on August 3, 2004, from Cape Canaveral Air Force Station in Florida, USA. It was the first spacecraft to enter Mercury's orbit. The mission was initially planned for 8 years, but it was extended multiple times and ended in April 2015.

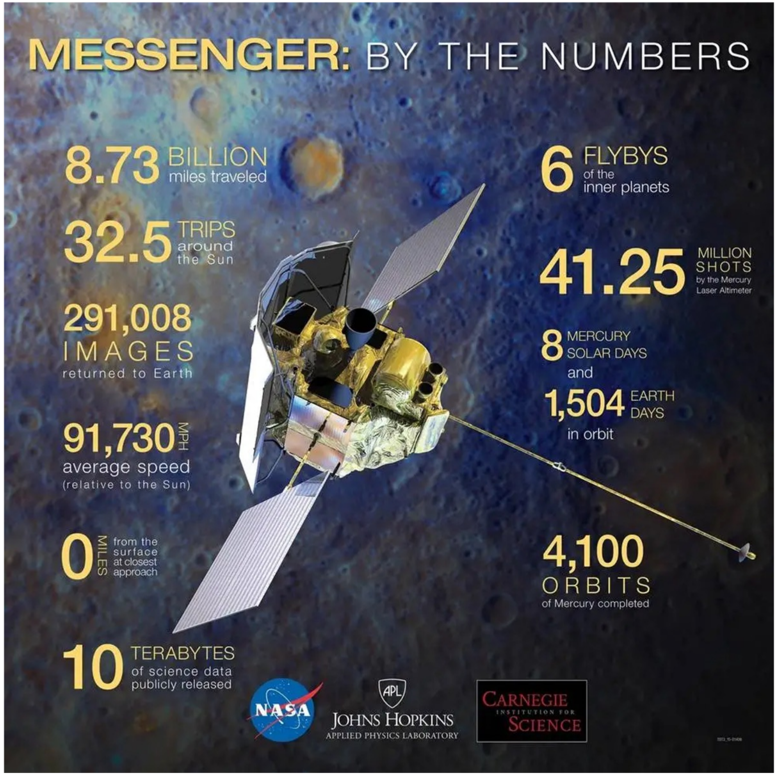

Figure 1 — MESSENGER spacecraft

Goal and objectives

Relevance: High-precision ephemerides of the Sun and planets play a critical role in modern solar system research.

Goal: Calculation and refinement of the MESSENGER spacecraft's orbit.

Objectives:

1. Processing Doppler frequency shift raw data.
2. Modeling the MESSENGER spacecraft's orbit , including:
   a. A model of Mercury's gravitational field.
   b. A model of Mercury's solid-body tides.
   c. Ephemerides of perturbing celestial bodies (Sun, Moon, and planets).
3. Computing model values of the Doppler frequency shift.
4. Refining the spacecraft's orbital parameters using the Gauss-Newton method.

Processing Doppler frequency shift raw data

- Orbit Data Files (ODF) is used to determine the spacecraft's trajectory, the gravitational field acting on it, and radio signal propagation conditions.
- An ODF file is a standard binary file containing multiple 36-byte records, which are divided into 4 main groups.
- The pds4-tools library was used to read ODF files in the Python 3 programming language.

Processing Doppler frequency shift raw data

The Doppler effect refers to the change in frequency and, consequently, the wavelength of a signal perceived by an observer due to the motion of the signal source relative to the observer.

Data: One-way, two-way, and three-way Doppler frequency shifts.

Mission phase	Begin date dd-mm-yyyy	End date dd-mm-yyyy	Number of 2-way Doppler	Number of 3-way Doppler	Number of range
Prime	17-05-2011	18-03-2012	2108980	184138	11540
Extended	26-03-2012	18-09-2012	1142974	23211	5709

Figure 2 — Number of observable data points.

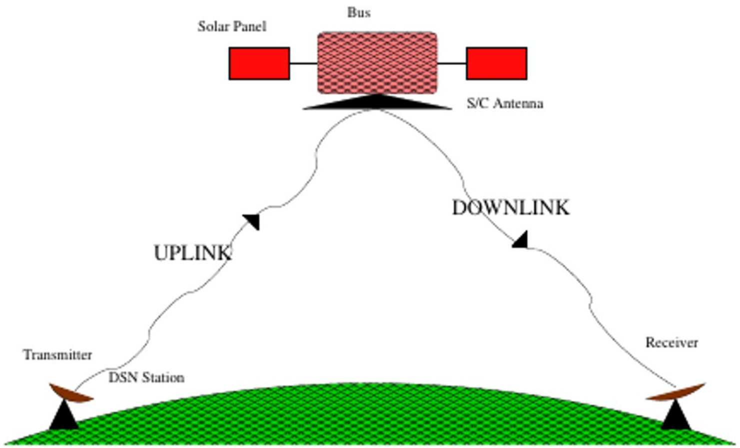

Figure 3 — Illustration of the signal transmission process.

Processing Doppler frequency shift raw data

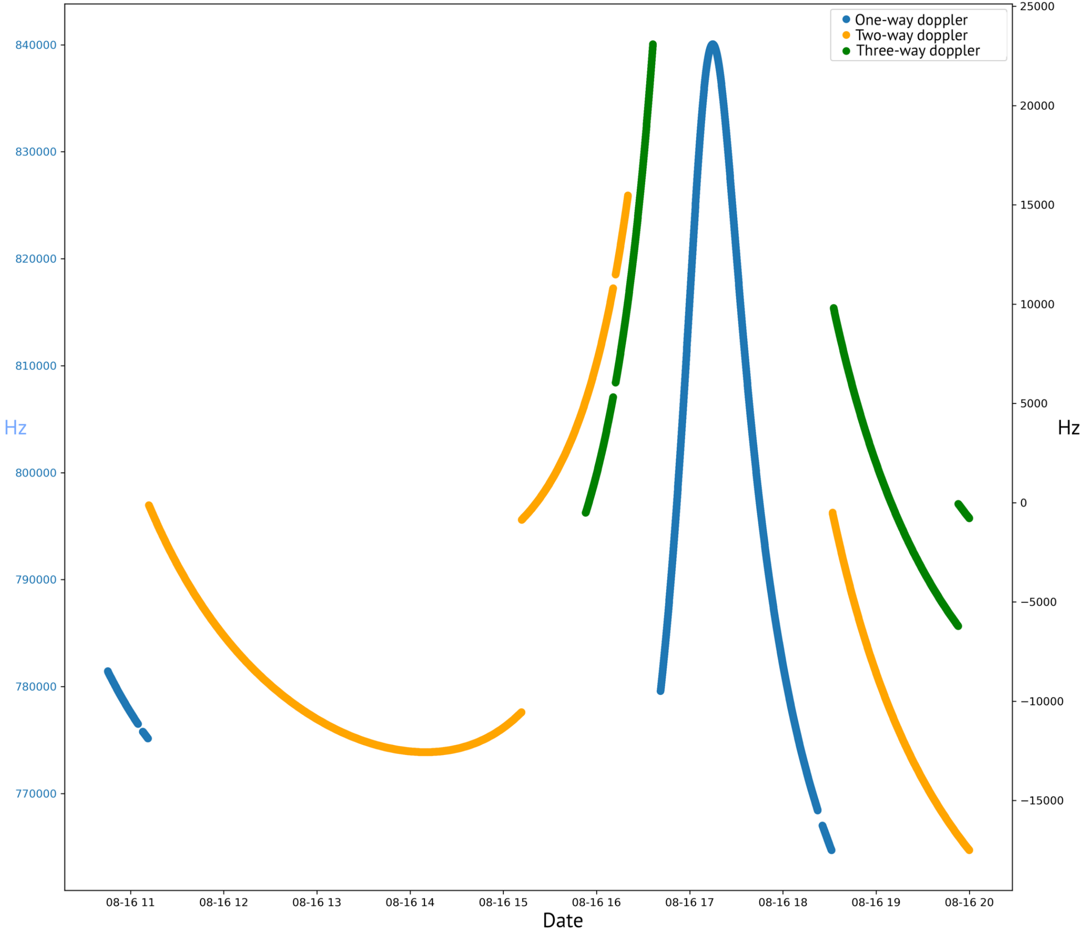

Figure 4 — Doppler frequency shift plot.

Modeling the MESSENGER spacecraft's orbit

The positions and velocities of perturbing bodies were obtained from the DE440 ephemerides.
The dynamic model included perturbations caused by:

- The Sun, Moon, and planets, as well as relativistic corrections.
- Mercury's gravitational potential.
- Mercury's solid-body tides.

The ABMD integrator was used in this work, implementing a multistep Adams-Bashforth-Moulton 13th-order predictor-corrector numerical integration scheme.

For modeling Mercury's gravitational potential, normalized spherical harmonics coefficients up to degree and order 6 (inclusive) from ggmes_100v08 were used.

Most computations were performed using double-double precision numbers (double-double arithmetic).

Modeling the MESSENGER spacecraft's orbit

Perturbations caused by the Sun, Moon, and planets, as well as relativistic corrections, are described by the Einstein–Infeld–Hoffmann equation:

$$\begin{aligned}
\vec{a}_A = &\sum_{B\neq A} \frac{Gm_B\vec{n}_{BA}}{r_{AB}^2} \\
&+ \frac{1}{c^2}\sum_{B\neq A}\frac{Gm_B\vec{n}_{BA}}{r_{AB}^2}\left[v_A^2 + 2v_B^2 - 4(\vec{v}_A\cdot\vec{v}_B) - \frac{3}{2}(\vec{n}_{AB}\cdot\vec{v}_B)^2\right. \\
&\qquad \left. - 4\sum_{C\neq A}\frac{Gm_C}{r_{AC}} - \sum_{C\neq B}\frac{Gm_C}{r_{BC}} + \frac{1}{2}((\vec{x}_B - \vec{x}_A)\cdot\vec{a}_B)\right] \\
&+ \frac{1}{c^2}\sum_{B\neq A}\frac{Gm_B}{r_{AB}^2}\left[\vec{n}_{AB}\cdot(4\vec{v}_A - 3\vec{v}_B)\right](\vec{v}_A - \vec{v}_B) \\
&+ \frac{7}{2c^2}\sum_{B\neq A}\frac{Gm_B\vec{a}_B}{r_{AB}}
\end{aligned}$$

x_A, v_A, a_A — barycentric position, velocity and acceleration of body *A;*
r_{AB} — distance between body A and B;
n_{AB} — normalized vector from body B to body A;
m_A — mass of body A;
c — speed of light;
G — gravitational constant.

Computing model values of the Doppler frequency shift

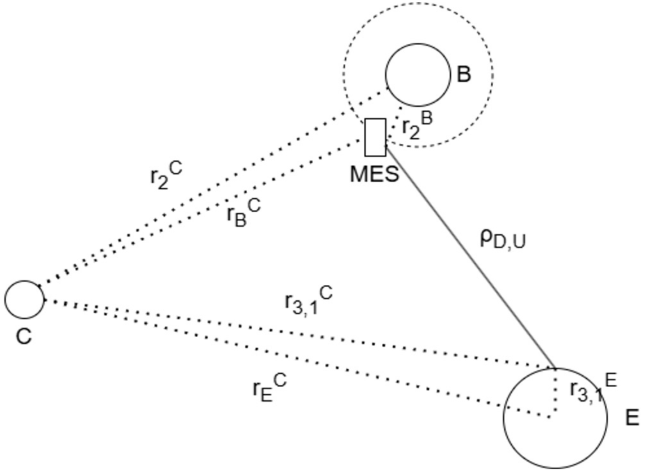

Figure 5 — Relative position of bodies.

The observed data gives the time t_3 of receiving the DSN signal. To calculate the Doppler shift, we need the time t_1 the signal was sent by the DSN station.
Below is an algorithm for finding t_2 from t_3, finding t_1 from t_2 occurs using similar procedure.

$$r_2^C(t_2) = \left[r_B^C(t_2) + r_2^B(t_2)\right]_{t_2=t_3} \quad (1)$$

$$\tau_D \approx \frac{1}{C}\left[\left|r_2^C(t_2) - r_3^C(t_3)\right|\right]_{t_2=t_3} \quad (2)$$

$$t_2 = t_3 - \tau_D \quad (3)$$

$$\rho_D = r_2^C(t_2) - r_3^C(t_3) \quad (4)$$

$$\tau_D = \frac{1}{C}(|\rho_D|) \quad (5)$$

To refine the value of t_2 calculations (3 – 5) should be repeated several times.

${r_2}^C$ — position of MESSENGER relative to the Sun at time t_2; ${r_B}^C$ — position of Mercury relative to the Sun; ${r_2}^B$ — position of MESSENGER relative to Mercury at time t_2; $\rho_{D,U}$ — distance between the DSN station and MESSENGER during "downlink" and "uplink" signal transmission; ${r_{3,1}}^C$ — position of the DSN relative to the Sun at times t_3 and t_1; ${r_E}^C$ — position of Earth relative to the Sun; ${r_{3,1}}^E$ — position of the DSN relative to Earth at times t_3 and t_1; $\tau_{D,U}$ – signal delay for "downlink" and "uplink”.

Computing model values of the Doppler frequency shift

The two-way and three-way Doppler shifts are calculated as the difference between accumulated Doppler cycle counts:

$$F_{2,3} = M_2 f_T(t_3) - \frac{M_2}{T_c} \int_{t_{1s}}^{t_{1e}} f_T(t_1) dt_1$$

M_2 — frequency gain factor; $f_T(t_3)$ — frequency of the signal received by the DSN station at time t_3; T_c — signal "compression" time; $f_T(t_1)$ — frequency of the signal transmitted by the DSN station at time t_1.

Refining the spacecraft's orbital parameters using the Gauss-Newton method

The difference between the observed value and the model value is called a residual

If there are m functions $r = (r_1, \ldots, r_m)$ (residuals) of n variables $\beta = (\beta_1, \ldots, \beta_n)$, with $m \geq n$. The Gauss — Newton algorithm iteratively finds the values of the variables that minimize the sum of squares:

$$S(\beta) = \sum_{i=1}^{m} r_i^2(\beta)$$

Refining the spacecraft's orbital parameters using the Gauss-Newton method

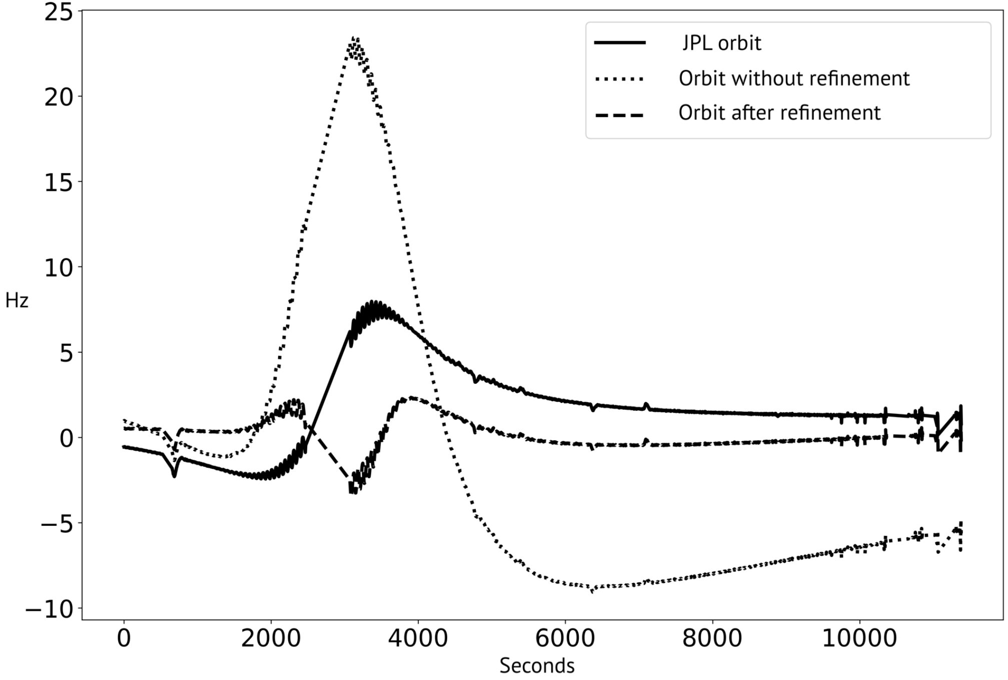

Figure 6 — Doppler shift residual plot (DSN №26)

System parameters — initial position and velocity of the MESSENGER spacecraft.

Table 1 — Root mean square deviation between model and observed doppler frequency shift values, Hz

Iteration	$\sqrt{\frac{\sum_{i=1}^{m} r_i^2}{m}}$
0	6.021
1	43.618
2	2.352
3	0.586
4	0.544

Conclusion

- For the first time, the raw radiometric data of the MESSENGER spacecraft have been processed independently from international researchers.
- A high-precision orbit of the MESSENGER spacecraft has been constructed, accounting for the perturbing effects of the Sun, Moon, and planets, Mercury's gravitational potential model, and solid-body tidal effects.
- The initial orbital parameters of the MESSENGER spacecraft have been refined, reducing the Doppler shift residuals.

Future work involves improving the Solar System model, particularly Mercury's orbit, and studying the dynamic properties of the Sun and fundamental properties of spacetime.