Correlation Analysis of Ranking Methods via Transitive Triads in Random Tournament Graphs

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1 Introduction

This study proposes alternative ranking methods aimed at improving fairness over simple win-count scoring. We introduce a new scoring approach and analyze the correlation between this method and traditional win-count rankings in randomly generated tournament graphs, with particular attention to the roles of transitive and non-transitive triads. By examining the correlation, we assess whether the proposed method offers meaningful improvements over the conventional approach.

2 Introduction of Two Ranking Methods

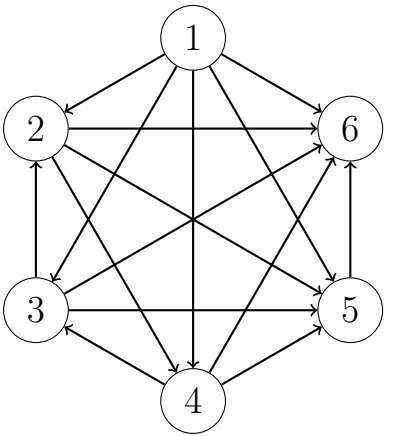
2.1 Method 1: Standard Ranking (SR)

- **SR points**: The total number of wins each player accumulates during the tournament.
- **SR ranking points**: Assigned based on the ranking of players by their SR points. The player with the most wins gets n ranking points, the next gets $n-1$, etc. In case of a tie, players receive the average of the corresponding ranking positions.

Example:

- 6 players: 1, 2, 3, 4, 5, 6
- Wins (SR points): 5, 3, 3, 3, 1, 0
- SR ranking points: 6, 4, 4, 4, 2, 1

Complete Tournament (6 players)



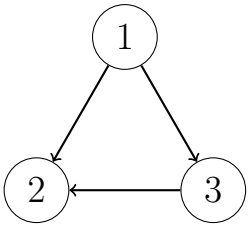
2.2 Method 2: Alternative Ranking (AR)

- Players form all possible subsets of size $k \geq 2$ (i.e., subtournaments)
- For each subset $S \subseteq P$:
  1. Virtual tournament is conducted within S using actual match outcomes
  2. Players in S are assigned SR ranking points based on performance in S
  3. Denote subset points for player p as $r_S(p)$
- Player's total score:

$$R(p) = \sum_{\substack{S \subseteq P \\ |S| \geq 2 \\ p \in S}} r_S(p)$$

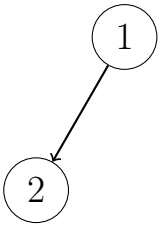
- Apply SR method again to $\{R(p)\}$ to determine final AR ranking points

Subset (Complete graph): {1,2,3}



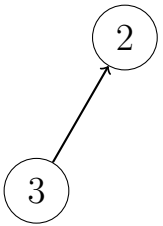
SR Points:
2, 0, 1
SR Rank Points:
3, 1, 2

Subset: {1,2}



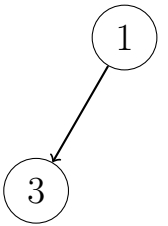
SR Points:
1, 0
SR Rank Points:
2, 1

Subset: {2,3}



SR Points:
0, 1
SR Rank Points:
1, 2

Subset: {1,3}



SR Points:
1, 0
SR Rank Points:
2, 1

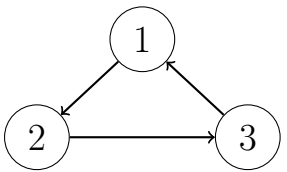
3 Research Plan

- Generated 1,000,000 random tournaments
- Used 4 modification types for triads
- Calculated rankings using two methods
- Analyzed correlations

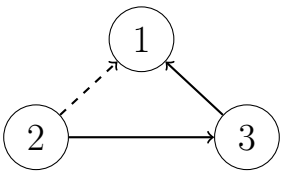
4 Methodology

We study how changing non-transitive triads affects ranking correlation. There are 4 possible ways to flip edges in a triad:

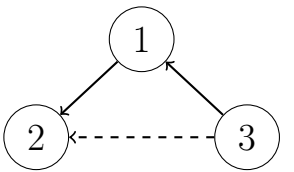
Original Triad



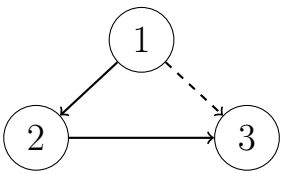
Type 1



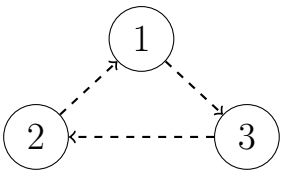
Type 2



Type 3



Type 4 (All edges flipped)



5 Experimental Data

The following tables present the statistical data for each correlation method used in our experiments:

Correlation Method	Mean	Variance	Standard Deviation
Spearman's rank correlation coefficient	0.965954	0.00111474	0.0333877
Kendall's tau coefficient	0.965954	0.00111474	0.0333877
Modified Kendall's tau	1	0	0
Custom Metric 1	0.843738	0.00758343	0.0870829
Custom Metric 2	2.93272	0.103837	0.322237

Table 1: Statistical Data for Correlation Methods

6 Conclusion

This paper proposes an alternative ranking method (AR) based on subset scoring, aiming to improve the fairness of traditional win-count-based ranking (SR) in the presence of non-transitive structures.

Through experiments on one million randomly generated tournament graphs, we found that although AR and SR rankings exhibit a high overall correlation (Spearman ≈ 0.966), AR is more sensitive to local structural variations. This suggests that AR has potential advantages in handling complex interaction patterns.

However, while AR may offer advantages in such cases, its computational cost is also significantly higher. Future research will focus on analyzing where these advantages are most evident and exploring ways to reduce the computational complexity of the second method.

Thanks for your listening

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