About the Implementation of Bi-directional Topology Optimization in ACELAN-COMPOS Package

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Abstract. The module of topological optimization in the ACELAN-COMPOS package based on the finite element method is considered. Optimization is carried out by a bidirectional evolutionary method of topological optimization. The module is tested on model problems.It can be used to conduct a series of numerical experiments aimed at creating effective designs of piezoelectric transducers.

Introduction

Design optimization is a relatively new but actively developing area of research that covers a wide range of tasks aimed at increasing the functionality and efficiency of the designed objects. Within this direction, three key types of tasks are distinguished: optimization of dimensions, shape and topology. Latter is the most complex and fundamental task.

Topological optimization uses algorithms to find the best material distribution in a given design area. This process is based on preset boundary conditions and a set of constraints, such as loads, mechanical properties of the materials, or allowable deformations. The final result is usually intended to maximize system performance or minimize costs while maintaining or improving functionality [1].

1. Mathematical modeling

The ACELAN–COMPOS package is a finite element software package designed to solve problems with coupled fields, such as electroelastic problems. The behavior of elastic and electroelastic bodies is described by the following equations and constitutive relations:

$$\begin{array}{ll} \rho_j \omega^2 \ddot{\boldsymbol{u}} + -\nabla \cdot \boldsymbol{\sigma} = \boldsymbol{f_j}; & \nabla \cdot \boldsymbol{D} = 0; \\ \boldsymbol{\sigma} = \boldsymbol{c_j^E} \cdot \cdot \boldsymbol{\varepsilon} - \boldsymbol{e_j^T} \cdot \boldsymbol{E}; & \boldsymbol{E} = -\nabla \varphi; \\ \boldsymbol{D} = \boldsymbol{e_j} \cdot \cdot \boldsymbol{\varepsilon} + \boldsymbol{k_j} \cdot \boldsymbol{E}; & \boldsymbol{\varepsilon} = (\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)/2 \end{array} \tag{1}$$

where $\boldsymbol{\sigma}$ — stress tensor, ρ — density, $\boldsymbol{\varepsilon}$ — deformation tensor, $\boldsymbol{u}$ — displacement vector, $\boldsymbol{D}$ — electric induction vector, $\boldsymbol{E}$ — electric field strength vector, $\boldsymbol{f}$ — vector of body forces, φ — electric potential, $\boldsymbol{c}^E$ — tensor of elastic constants, $\boldsymbol{e}^T$ — piezoelectric tensor, $\boldsymbol{k}$ — permittivity tensor, ω — circular frequency, $\cdot\cdot$ is a tensor product, j — body number in the model.

The topology optimization module in the ACELAN–COMPOS package is implemented based on evolutionary methods (ESO, BESO) [4]. The Evolutionary Structural Optimization (ESO) method [3] is to remove excess material from the structure. An indicator of inefficient use of the material is the low level of stress (or deformation) in this part. Preferably, the material distribution is such that the stress level in the structure is close to the limit but safe value and evenly distributed. This implies the principle of material removal, according to which insufficiently loaded material can be removed. In order to avoid rearrangement of the finite element mesh, material removal is carried out by assigning the removed element the effective properties of a pseudo-material that simulates emptiness.

When optimizing the topology of complex structures, it is important to consider the possibility of returning previously removed material. This possibility is provided by the method of bidirectional evolutionary optimization BESO. In this method, deleted items with the highest sensitivity values are returned to the system, and filled items with the lowest sensitivity values are deleted. The number of items to be removed and added on each iteration is determined by two independent parameters: removal factor and inclusion factor [2].

To optimize the topology in the ACELAN–COMPOS package, it is necessary to have a model of the device and data for optimization. Deformations, stresses and displacements can act as a function of sensitivity assessment. The optimization process runs until the limit is reached or the maximum number of optimization steps is reached. Each optimization step consists of several steps.

At the first stage of each step, direct problems are solved for each set of boundary conditions that describe the loading scheme of the model, and the sensitivity values of the elements are found. Deformations, stresses or displacements can act as a function of sensitivity assessment.

When meshing a continuous structure into finite elements, the sensitivity function may allow a discontinuity at the boundaries of the elements, forming a topology in the form of a "checkerboard". To avoid this, smoothed sensitivity of elements with filtering is used [2]. By smoothing the sensitivity values over the entire computational domain, removed elements are automatically assigned non-zero sensitivity values.

The second step calculates the constraints and the goal of topology optimization. If a target or constraints are found, samples are taken for the model.

Samples can be: body volume, body weight, average voltage and other integral characteristics. Constraints can be evaluated from above or below.

To optimize BESO, the next step is to return the removed material. The original material is returned for the removed elements with the maximum sensitivity value.

Next, the elements are deleted, and it is assumed that the active load is not affected by the element deletion. Features containing nodes that have boundary conditions applied cannot be deleted. The material of the elements with the minimum sensitivity values is replaced with a pseudo material for removal, if the model connectivity to the first material is not broken when this element is removed. Before removing or returning material, a check is made for each element to be removed to see if the number of FEM connectivity components changes.

The result of the optimization in the ACELAN–COMPOS package is a set of files for each optimization step: grids in .nas format for export to outside packages and in its own .jam format based on JSON syntax for display in ACELAN–COMPOS. The JAM file for each set of boundary conditions includes nodal, elemental, and model solutions (objective functions and constraint models). Entire functions and element availability information are stored for full process optimization and transformation at each stage.

2. Numerical experiments

A model of a device made of PZT-4 ceramics is considered. The height and width are 10 *mm*, and the length is 16 *mm*. The left face is fixed, a load is applied to the center point of the right face. To obtain a symmetric solution, the load is directed in two directions along the Z axis.

During the study, the problem of topological optimization was solved by the BESO method. Optimization is carried out before the removal of 80% of the material, in one step 1% of the elements are removed and 0.5% elements are restored. Isotropic material with properties close to air is used as material for removal. The maximum averaged stresses act as the optimization objective function, and the body volume and maximum averaged strains act as constraints.

The ACELAN-COMPOS package implements linear and quadratic cubic finite elements and linear tetrahedral finite elements. The finite element mesh of tetrahedrons and the obtained optimization results are shown in the figure 1.

Such resulting topology is not yet suitable for practical purposes because the mesh has sharp edges on the surface. To solve this problem, it is most efficient to use an additional shape optimization step which changes the position of the partition points on the surface. Their positions change only along the normal to the surface. As a result, smoother structures are obtained and they are easier to produce. In addition, potential stress concentrators on the surface are eliminated.

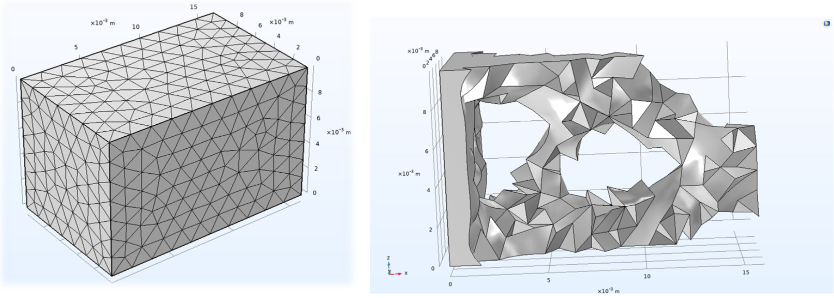

Figure 1. Finite element mesh of model and optimization results with 80% material removed

Conclusion

The module for topology optimization in the ACELAN–COMPOS package, implemented on the basis of evolutionary optimization methods, is considered. The developed optimization module can be used to conduct a series of numerical experiments aimed at creating effective designs of piezoelectric transducers.

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