The Use of Smoothed Distribution Functions for the Study of Chaotic States in Nonlinear Dynamics Problems: A Numerical Experiment

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Abstract. The possibility of using distribution functions to describe chaotic states in nonlinear dynamics problems is considered. The system of equations has the form:

$$\dot{\mathbf{r}} = \mathbf{v}(\mathbf{r}) \tag{1}$$

where r is a vector representing a set of variables of a given system, and v(r) is a vector function of these variables. Such distribution functions determine the probability density of finding a system in a certain region of the phase space and satisfy a first-order partial differential equation:

$$\dot{\rho}(\mathbf{r},t) + \mathrm{div}\big(\mathbf{v}\rho(\mathbf{r},t)\big) = 0 \tag{2}$$

It is shown that in the case of dissipative systems, when the state of the system tends to a state described by a chaotic (strange) attractor with increasing time, it is more appropriate to use smoothed distribution functions obtained by averaging over some small regions in the phase space. The resulting equation for smoothed distribution functions has the form:

$$\dot{\overline{\rho}}(\mathbf{r},t) + \mathbf{v}\cdot \mathrm{grad}\big(\overline{\rho}(\mathbf{r},t)\big) = 0 \tag{3}$$

This "smoothing" well correspond with numerical calculations, which always use finite cells to solve differential equations. The resulting equation was used for numerical calculations of the radiation intensity of a number of specific systems in a state of chaos, in particular, the Duffing equation, which is usually used to demonstrate the appearance of chaotic states in nonlinear dynamics problems. In this case, the radiation was described as a consequence of fluctuations in smoothed distribution functions.

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