Effective Solution of the Traveling Salesman Problem on Randomly Distributed Data

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Abstract. In this study, we present a method and its implementations for the traveling salesman problem (TSP) considering matrices with a uniform random distribution of path costs. The method is a modification of the branch-and-bound method with specific heuristics. Data generation and validation used for this study are discussed. Core assumptions and heuristics used in the method's implementations are provided. Numerical results, including comparisons with cycle merging algorithms, are presented. The suggested algorithm was compared with existing implementations and showed better accuracy by finding cheaper Hamiltonian cycles for a majority of synthetic problems generated for this study. Results are promising for applications with limited sizes of TSP problems (up to 100 nodes); the method is suitable for consumer-grade hardware while maintaining the accuracy of results.

Introduction

The traveling salesman problem (TSP) arises in many practical applications and has a long history of method development [1, 2]. It holds significant importance for computational mathematics and graph algorithms since it is an NP-hard problem [3]. The problem can be described in the following way: For a set of nodes $V = 1, 2, \ldots, n$ we have a distance matrix $D = [d_{ij}]$ where d_ij represents the cost (distance, time or any other measure) of traveling from node with number i to node with number j. The following limitations apply to the values of matrix D:

$$\begin{cases} d_{ij}0, & i \neq j \\ d_{ij} = +\infty, & i = j \end{cases}$$

The objective is to find a Hamiltonian cycle, i.e., a closed tour visiting each node exactly once with minimal total travel cost.

In this work, we implemented the method for a general random non-symmetric distance matrix. Practical applications, such as drone delivery and Prize-Collecting

TSP, often involve near-random data [4, 5]. We present a branch-and-bound method with specific heuristics for solving the TSP on randomly distributed data, and compare its performance to the cutting plane method.

1. DATA GENERATION AND METHOD DESCRIPTION

The data is generated using pseudo-random number generators for matrices of size 49 and 99. Distances varied from 0 to 999, and diagonal elements are set to 999999 to represent the infinite cost of self-loops. The data was validated to be uniformly distributed using the chi-squared goodness-of-fit test. Expected frequency for each possible value in a discrete uniform distribution is

$$E = \frac{N}{1000}$$

where N is a number of generated values. The chi-squared statistics were computed using the formula

$$\chi^2 = \sum_{i=0}^{999} \frac{(O_i - E)^2}{E}$$

where O_i is the observed frequency of value i, and E is the expected frequency. The resulting statistic was compared against the chi-squared distribution to determine the corresponding p-value. A p-value below the significance threshold would indicate a statistically significant deviation from uniformity. In this study, we used a threshold value of 0.05.

In previous works [6, 7, 8], we discussed different approaches to implementing effective solutions to the TSP problem. In this work, we implemented an algorithm based on our version of the branch-and-bound method with multiple heuristics.

2. NUMERICAL EXPERIMENTS

To compare the developed implementation with relevant modern approaches, we generated 100 test matrices: 10 with 99 rows and 90 with 49 rows. The Cycle Merging Algorithm [9] and a single-core implementation of the presented algorithm. In 90 tests, the presented algorithm provided a shorter cycle, and in the remaining 10 tests, the results were equal. Visual comparisons for 20 randomly selected tasks and for the entire dataset are presented in Figure 1. All computations were performed on the same hardware with a multi-core CPU (AMD Ryzen 9 7940H) with 32 Gb RAM.

In some cases, presented method halted on exceeding the maximum amount of steps, so the produced solution may be sub-optimal.

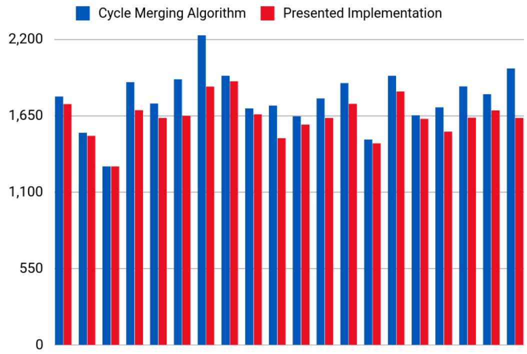

FIGURE 1. Comparison of the cost of the shortest obtained Hamiltonian cycle for cycle merging algorithm and presented implementation for 20 random problems with 99 nodes (left) and average, maximum and minimum cost of the shortest obtained Hamiltonian cycle (right).

3. CONCLUSION. POSSIBLE SUBJECT AREAS OF THE FURTHER TOPIC DEVELOPMENT

Our implementation of the modified branch-and-bound method shows good accuracy for random TSP problems in which no extra assumptions can be made. It outperforms modern solutions for problems of the moderate size with 50-100 nodes on consumer-grade computational hardware. In future work, we plan to parallelize the algorithm to improve its performance on large-scale problems.

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