The Role of Computers in Developing Mathematical Thinking in Young Children

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Abstract. This paper explores the role of computers in fostering mathematical thinking in young children, starting from the preschool age. While traditional education often equates early mathematics with numeracy and arithmetic skills, we argue that true mathematical thinking involves the ability and readiness to solve unfamiliar and challenging problems. Conventional schooling tends to suppress this ability by encouraging drill-and-practice and repetitive problem types. In contrast, algorithmic, computational thinking—promoted since the work of Seymour Papert—offers a richer and more authentic mathematical experience, even for very young learners. We advocate for learning environments grounded in visual and tangible representations of mathematical objects and operations, supported by digital tools. Such environments not only engage children meaningfully but also allow for customisation, feedback, and accessibility. We present examples from both early childhood and higher education settings, demonstrating how computers can visualise core mathematical structures—such as symbol strings, multisets, tables, and trees—and enable deep conceptual learning through interaction, construction, and reflection.

Keywords: mathematical thinking, early childhood education, computer-based learning, visual representations, algorithmic thinking, preschool mathematics, educational technology, microworlds, Logo, PiktoMir, personalised learning

Introduction

We often speak about the importance of developing mathematical reasoning, thinking starting in primary school—and, perhaps more controversially, or metaphorically, even before school.

Yet, whenever the conversation proceeds toward specifics, the topic quickly narrows to terms like *numeracy,* arithmetic skills, and dyscalculia. But does this truly constitute mathematical thinking? We believe it is true only to a very limited extent. While mathematical thinking can be developed through arithmetic objects and skills, this is difficult due to their abstract and non-visual nature—and in practice, it rarely happens. This has been clearly stated by prominent figures such as V.I. Arnold [1], A.Ya. Khinchin [2], V.V. Firsov [3], and many others.

At the same time, since the pioneering work of Seymour Papert, we have become accustomed to the idea that "programmer's thinking" or algorithmic thinking can be nurtured from early childhood. Today, this is being done remarkably well by Anatoly Kushnirenko's group in mainstream kindergartens —for example, in all kindergartens in Surgut. It is evident that such "programmatic thinking" often involves far more genuine mathematical reasoning than traditional school arithmetic!

In our view, the essence of mathematical thinking lies in the ability and willingness to tackle unexpected, or challenging problems. Remarkably, even a newborn baby exhibits this quality to some degree.

Unfortunately, formal education tends to suppress this natural ability. School arithmetic, in particular, teaches pupils to memorise methods for executing of few specific symbol manipulation algorithms and then assesses their learning using near-identical, formulaic exercises. This approach, we see, is the core flaw of standardised testing such as the Unified State Exam (EGE).

But in today's world, there is no longer any need to train children to rapidly and flawlessly imitate an electronic calculator.

1. So how should we nurture the essential ability to solve un-expected problems?

We must offer children a visual—and ideally also tactile—world of objects, their properties, and the operations that can be performed on them. Such experiential environments were championed by Froebel, Montessori, Cuisenaire, and Papy. Alexander Zvonkin [4] also wrote compellingly on this theme. Many of the *Kangaroo* mathematics competition scenarios developed by Mark Bashmakov grow out of such worlds.

Digital equivalents include Papert's microworlds (LOGO), and for even younger children: *PervoLogo* by A. Semenov and S. Soprunov [5], and *PiktoMir* by A. Kushnirenko [6], and developments of Mitch Resnik group: *Scratch* [7], followed by *Octopus* and *Snap!* [8, 9] by Jens Mönig and Brian Harvey.

In higher education, significant progress in cultivating mathematical thinking has also been achieved—thanks to the computer—through the work of Nikolai Vavilov [10].

Our central idea is this: computers can give concrete, visual (on-screen) form to basic concepts from (finite) mathematics.

We include among basic objects [11]:

- Strings of symbols
- Bags (multisets)

- Tables
- Trees and their natural operations

Simultaneously, names of objects and logical operations such as "for all" and "there exists" emerge organically. From there, come programmes, games, and strategies. Crucially, all objects, actions and processes relevant to a task are visible at once —on a screen or a page. Small numbers (up to 10 or 20) pose no problem—they are useful and can be represented by rectangular areas, making operations on them visually intuitive. Larger numbers will arise naturally when the child, with guidance, invents their own counting algorithms.

The computer (or tablet) plays a crucial role in the following ways:

- *Saving paper* – Both economically and psychologically, the idea of paper-saving still presents a challenge, especially when it comes to printing educational materials for children. Computers help overcome this issue entirely.
- *Reading problem statements aloud* – This is particularly important for preschoolers who are not yet confident readers. It is especially valuable for children from families non-speaking language of instruction, where features such as translation into the child's native language can also be considered.
- *Personalised settings* – For example, the environment can be restricted to make tasks more challenging, or, conversely, hints and prompts can be enabled when needed, even allowing the option to reveal the answer if necessary.
- *Recording all of the child's actions* – This allows for later analysis, feedback, tailored recommendations, and progress forecasting. It can definitely displace exams.

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