Factorial Numbering in Combinatorics: Comparative Analysis of Pedagogical and Computational Implementations

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Abstract. This article compares two permutation generation algorithms based on factorial numbering: Derrick H. Lehmer's computational method (1964) and Charles-Ange Lesant's pedagogical approach (1888). Despite mathematical equivalence, they represent distinct historical-methodological constructs. Lesant's algorithm prioritizes educational clarity and rigor within natural number theory, featuring step-by-step manual execution and foundational examples. In contrast, Lehmer's algorithm solves the practical problem of efficient permutation generation on early computers, emphasizing operational optimization and table-based (sieve-like) implementation suited for punched cards. By contrasting their goals (educational accessibility vs. computational pragmatism), justification methodologies (conceptual analysis vs. efficiency-driven design), and implementations (manual vs. machine-oriented), the study confirms their fundamental divergence. The work illustrates how identical combinatorial structures serve distinct historical purposes: advancing pedagogical frameworks and technological methodologies.

Introduction

The development of mathematical algorithms is closely related to the historical, cultural and technological conditions of their creation. This article is devoted to the comparison of two algorithms for generating permutations using the factorial number system: the pedagogical approach of Charles-Ange Lesan (1888) and the computational method of Derrick Henry Lehmer (1964). Despite their mathematical similarities, these algorithms pursue different goals and reflect different eras. Lesan sought to make mathematics accessible to a wide audience by creating a clear and understandable tool for teaching combinatorics. Lehmer solved the practical problem of efficiently generating permutations on early computers, responding to the technological challenges of his time. By comparing their goals,

methodologies, implementations, and historical contexts, this work shows how the same mathematical structures can serve different educational and computational tasks, revealing the evolution of mathematical thought.

1. Background

The Lesan algorithm was developed as a pedagogical tool for demonstrating the factorial number system and combinatorial tools. This is one of the elements of the article aimed at popularizing the accessibility of general mathematics education. The Lehmer algorithm was created to solve the practical problem of implementing and optimizing the generation of permutations on early computers, focusing on computational efficiency and demonstrating the implementation of standard methods for constructing combinatorial tools.

2. Methods

The comparative analysis uses a historical and methodological approach based on a detailed study of the original works of Lesan and Lehmer. The comparison criteria — the goals of creation, the methodology of justification, the specifics of implementation and the results — were selected as system-forming to identify the essential differences in the application of factorial numbering. In "goals" analyzes the context of the emergence of algorithms, "justification methodology" — differences in argumentation (rigorous evidence vs operational efficiency), "implementation" — adaptation to the execution environment, "results" — achievement of the goals stated by the authors. This structure makes it possible to compare the systemic methodological paradigms of algorithms, which is critical for understanding their role in the history of mathematics and computer science.

3. Results

The article analyzes fundamental differences in methodological approaches: the Lesan factor numbering serves as a pedagogical tool for presenting combinatorial concepts, while the Lehmer transformation turns it into a combinatorial analysis tool optimized for the architectural limitations of early computers. It is shown that the key difference between the implementations is due to the target audience: Lesan's redundant steps and terminology ("selection mécanique") are focused on pedagogical clarity, while Lehmer's concise tabular structure reflects the features of machine data processing. It is demonstrated that both algorithms, despite their isomorphic mathematical basis, form various historical and scientific objects.

4. Discussion on the importance of the work

This study resolves the historical terminological ambiguity that arose due to the formal similarity of the Lesan and Lehmer algorithms by comparing in detail their initial goals, justification methodologies, and creation contexts. The systematic analysis demonstrates how isomorphic combinatorial structures adapt to solve fundamentally different problems, reflecting the evolution of mathematical thought under the influence of technological progress and socio-cultural demands. The results obtained contribute to the history of mathematics, clarifying the genesis of ideas and sharing priorities in the development of combinatorial methods, as well as to the methodology of computer science, emphasizing the dependence of the format of algorithmic solutions on the instrumental and conceptual framework of the era. For modern pedagogy, the work illustrates the value of contextualizing mathematical concepts, showing how a single tool can serve both educational purposes and applied engineering tasks. Finally, the proposed scheme for comparing algorithms as historical and methodological artifacts offers a template for analyzing similar cases in the history of science and technology.

Conclusion

A comparative analysis of the Lesan and Lehmer algorithms revealed their fundamental differences, despite using the same mathematical basis — the factorial number system. The Lesan algorithm, developed in the context of the educational reform of France in the 19th century, served as a pedagogical tool aimed at clarity and accessibility of combinatorial ideas for students. On the contrary, the Lehmer algorithm, created during the era of computer technology development in the United States in the 1960s, was focused on optimizing computing for early computers, which emphasizes its practical importance. This work not only clarifies the unique contributions of Lesan and Lehmer, but also demonstrates how historical and technological factors shape mathematical ideas. The findings contribute to a deeper understanding of the evolution of mathematics and its impact on education and technology, providing a foundation for further research in the history of science and computer science.

References

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